Uniform Price Auctions with Budget Constrained Bidders: An Experimental Study on Budget Rules

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Abstract

The uniform price auction often allows financially constrained bidders to bid on, with auctioneers putting restrictions on bids to prevent bidders to pay a price that they cannot afford. For example, Google's initial public offering (IPO) and Korea's emission trade system auctions restrict bidders from submitting bids that add up to more than the amount of money that they can pay. In this study, I suggest an alternative budget rule that still maintains bidders' budget constraints and experimentally show that the alterative rule can improve the efficiency and revenue of auctions. The experiment simulated a weak budget and a tighter budget, and the improvement was much pronounced under the tighter budget.

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1. Introduction

If a seller sells a divisible item and bidders cannot afford the whole item on offer, selling shares of the item to many bidders can be more profitable than selling the whole item to a single bidder. The Uniform price auction has been widely used in such environments in which a seller sells multiple shares to bidders who alone cannot afford the entire shares, such as in initial public offering, treasury bill auctions and emission permit auctions. Phillips' Plan for Outer Continental Shelf (OCS) oil lease auctions is an old example where practitioners have suggested to use auctions like the uniform price auction to accommodate financially constrained bidders (Office of Minerals Policy and Research Analysis and Anderson, 1979, Wilson 1997). Since OCS tracts are often too large compared to bidders' ability to pay, Phillips' Plan suggested to allow bidders to submit bids on fractional working-interest shares and collect revenues from all bidders who receive any shares.

In the uniform price auction n identical shares are on offer and each bidder can submit bids on as many shares as they want. After all bids are submitted, the m highest bids each get a share and pay the market price, which is set by the $m+1$ highest bid. When a uniform price auction accommodates financially constrained bidders, the auction must incorporate a rule that prevents bidders from paying a price that they cannot afford. For example, when Google went to public using a uniform price auction, Google's IPO prospectus stated "*Do not submit bids that add up to more than the amount of money you want to invest in the IPO. This is a very important point*". Similarly, Korea's emission trading system (ETS) auction, which uses a uniform price auction to allocate emission permit, enforces a bidder to submit an upfront security fee that is equal to the sum of all bids that the bidder submits. Under this rule, a bidder with a total budget w cannot submit bids that add up to more than w.

The goal of this study is to show that the budget rule enforced by Google's IPO and Korea's ETS auction is unnecessary strong and to suggest an alternative rule to enforce budget. The alternative rule is to restrict each bidder's kth bid from exceeding w/k, which is a more relaxed restriction than the former rule while maintaining the overall budget constraint.[1] The paper then experimentally studies whether the alternative rule may increase the efficiency and the revenue of the uniform price auction.

The main setup of the experiment is to compare the efficiency and the revenue of the uniform price auction under the two different budget rules in the context of a private value auction[2]. For convenience, I refer to the uniform price auction with the conventional rule, used by Google's IPO, as UPA treatment and the alternative rule as UPA-M treatment[3]. In the experiment, a single supplier sells 5 shares of an item to 3 buyers. Each bidder is assigned a private value(v) for a share, which is drawn from a uniform distribution U[0,100]. Bidders have a common budget constraint w, the maximum amount of money that they can use for the shares on sale. The experiment was conducted with two difference levels of budget constraints. In the stronger budget constraint, the total budget was set by 100, whereas in the weaker budget constraint, the budget was set by 250.

Theory predicts that bidders will bid their true value on the first share under UPA-M treatment as it is a dominant strategy, while bidders in UPA treatment may bid less than the value on the first share to save budget and use it for the later shares. This poses a hypothesis that the efficiency would be higher in UPA-M as bidders submit bids closer to their values. In addition, as the budget rule is more stringent in UPA, this poses another hypothesis that UPA-M would achieves higher revenue than UPA.

The experimental outcomes are broadly consistent with the predictions of the theory and the hypotheses. Bidders in UPA-M more frequently submit truthful bids on the first share, and the

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[1] For example, suppose a bidder has a budget 100 and wishes to bid on three shares. Under the former rule the bidder can submit 100 on the 1st share, but then he has no more budget left for the other two shares. Under the alternative rule however, the bidder can bid 100 for the 1st share and 50 for the 2nd share and (approximately) 33 for the 3rd share. Yet, the bidder will never pay more than 100 under the second rule. For example, if he wins three shares, the market price must be equal or less than his third bid which is at most 33. Thus, his payment, (3 $\times$ the market price), will not exceed 100. This logic applies to whatever number of shares he gets.

2 In some applications, common value or affiliate value assumption would be more appropriate. However, the experiment was conducted in a private value environment to make the setting simple as the main goal of the experiment is to compare the efficiency and revenue implications between the two budget rules

[3] In a sense that the rule is modified.

efficiency and the revenue were higher in UPA-M treatment. However, the improvement was statically significant only when the budget constraint was strong (w=100). This shows that the relaxed budget rule effectively improves the outcome only when the budget is tight enough. Under the weaker budget treatment, the efficiency and revenue losses of the conventional rule compared to the alternative rule are both less than 3%, which supports the wide use of such budget rule in practice. Under both UPA and UPA-M, bidders tend to bid above their values, which is dominated strategy. However, bidders rarely ended up with a negative profit, which explains the dominated behavior to some extent.

The rest of the paper is organized as follows. Section 2 reviews the previous literature. Section 3 introduces the theoretical framework and hypotheses to investigate. Section 4 describes the experimental design and procedures, with the experimental results reported in Section 5. Section 6 discusses the results reported.

2. Related literature

The uniform price auction has been widely studied in the economics literature, but mostly *without* budget constraints. Wilson (1979), in response to the Philips Plans for OCS auction, showed that the uniform price auction would result in lower revenue than a single-unit standard auction. Ausbel et al. (2014) showed that bidders in the uniform price auction has a generic incentive to submit bids less than their value for the second and later units (=demand reduction), which could result in poor revenue performance such as zero-revenue equilibrium. Kagel and Levin (2001) demonstrated that experimental subjects indeed exercise demand reduction, the force behind the lower revenue prediction. List and Reiley (2000) also reported demand reduction behavior in a field experiment.

There have only been a few experimental studies of auctions that feature budget-constrained bidders. Pitchik and Schotter (1988) study two-stage sequential auctions in which two bidders can each strategically deplete the other bidder's budget in the first stage. Kotowski (2011, 2020) theoretically and experimentally studies the first price auction in which two bidders have private values and budgets. Ausbel et al. (2017) experimentally compare the first and second price auctions in which the budget is endogenously set by a financial manager. Bae and Kagel(2021) experimentally compare the efficiency and revenue between the first price auction and the proportional auction under budget constrained environment. This current paper is one of the first to experimentally study uniform price auction with budget constrained bidders along with associated budget rules..

3. Theoretical framework and experimental design

A seller offers m equal shares of an item without a reserve price and n bidders wish to buy the shares. All bidders are risk neutral and each bidder has a private value of the shares on sale. Let v_i be the value of a share for bidder i. If the bidder receives multiple shares, say x shares, the bidder's payoff equals to $v_i x$. Values are independently drawn from a distribution $F(v)$. For simplicity, bidders have a common budget constraint w, which is the most they can spend on the shares on offer.

In the uniform price auction (UPA), each bidder can submit up to m bids. After all bids are submitted, the bids are ranked from highest to lowest, and the m highest bids each get a share, with ties are randomly broken. The market price for a share is set by the $(m+1)th$ highest bid and bidders winning any number of shares pay the market price for each share they get.

The paper studies two different rules to enforce the budget constraint. The first rule is that bids must add up to no more than the budget constraint ($\sum_{k=1}^{m} b_i^k \leq w$). This rule is similar to Google's advice for their IPO: "*Do not submit bids that add up to more than the amount of money you want to invest in the IPO*". This rule can be practically implemented by asking security deposits for all bids submitted in a uniform price auction as Korea's emission permit auction requires. However, this rule is unnecessarily restrictive. An alternative rule that restricts each bidder's k^{th} bid from exceeding w/k is a more relaxed restriction but still satisfies the budget constraint.[4] I refer to the modified rule as UPA-M.

Characterizing an equilibrium in UPA (UPA-M) is involved and deferred to the appendix 1. [5] UPA (UPA-M) admits diverse multiple equilibria (Wilson, 1979; Engelbrecht-Wiggans and Kahn, 1998; Ausubel et al. 2014) and a closed-form expression for equilibrium bids is in general not available (Krishna, 2009). Thus, pinning down a unique equilibrium prediction is a difficult task.

However, Theories provide some predictions on equilibrium bidding behavior in UPA and UPA-M. First, bidding above values on any shares is a dominated strategy in both UPA and UPA-M. Second, bidders have generic incentive to submit bid lower than their values for the second and subsequence shares (Ausbel et al., 2014). Lastly, bidding true value on the first share is a dominant strategy in UPA-M but it is not in UPA (proof in the appendix 3). This is because the budget rule applies to the sum of all bids in UPA, so it could be more profitable to bid less than the value on the first share to save budget and use the saved budget to submit bids for later shares. In UPA-M, on the contrary, since the budget rule applies to each bid, bidders does not have incentive to lower bid on the first share to save budget.

The first two predictions are well known properties of the uniform price auction. In particular, the second prediction, which is referred to demand reduction, is widely studied in the auction literature both theoretically and experimentally (Ausbel et al, 2014; Kagel and Levin, 2001; List and Reiley, 2000). The third prediction is the most relevant to the current study as it predicts different bidding behavior between UPA and UPA-M treatment. Since the theory predicts truthful bidding on the first share in UPA-M but potential bid shaving on the first share in UPA, the difference implies that UPA-M may achieve higher efficiency and revenue than UPA. However, there is very

[4] See footnote 4

[5] In general, multi-unit auctions may not have an equilibrium with budget constraints. For example, Ghosh (2015) showed that *multi-unit simultaneous first-price* auction has *no* equilibrium in pure-strategy with a common budget constraint. In the appendix 2, however, I show an example in which UPA with a budget constraint has an equilibrium.

little empirical/experimental evidence supporting this claim and the current experiment is one of the first to study the efficiency and revenue implications of the two treatments.

The experiment adopts a 2 by 2 design with two auction formats (UPA, UPA-M) and two levels of budget constraints (w=250 and w =100), which simulate a weak and a strong budget. In each auction, three bidders (n=3) participate in the auction, and each bidder is assigned a private value v which is independently drawn from a uniform distribution on (0, 1, 2, …, 100). There are five shares on offer and bidders can submit up to five bids.

Three closely related hypotheses will be investigated in the study. First hypothesis is that bidders will more frequently bid the true value in UPA-M than UPA. This hypothesis is supported by the theory for the first share but theories do not predict truthful bid for the second and subsequent shares. However, truthful bidding for second shares is often employed by bidders in experimental studies (Kagel and Levin, 2001; List and Reiley, 2000), and more stringent budget rule in UPA may prevent such bidders from submitting the true value. The second hypothesis is that bidders in general will be more frequently constrained by the budget rule in UPA than UPA-M. This is simply because the budget rule is more restrictive in UPA. The third hypothesis is that UPA-M will achieve higher efficiency and revenue than UPA. If the first and second hypotheses turn out to be true, then as a consequence, the third hypothesis is very likely to be hold; more frequent truthful bidding would lead to higher efficiency and loosened budget rule would lead to higher revenue in UPA-M.

4. Experimental procedures

A total of 6 sessions were run, 3 sessions for each auction format. Between 15 and 21 subjects were in each session[6]. Subjects participated in 10 auctions under w=100, followed by 10 auctions under w=250. In each auction, 5 shares of an item were offered and subjects were assigned integer

[6] UPA sessions had 21, 15, 21 subjects in each session, and UPA-M sessions had 21,18,18 subjects in each session. A different set of subjects participated in each experimental session.

values randomly drawn from [0,100] with subjects being randomly rematched into different groups in each auction. The subjects could submit up to five integer bids (including 0) but were not mandated to submit all 5 bids. In UPA, the sum of bids could not exceed the budget constraint. In UPA-M, under w=100, subjects could submit up to (100, 50, 33, 25, 20) for (1^{st}, 2^{nd}, 3^{rd}, 4^{th}, 5^{th} share), respectively. Under w=250, the restriction was (250, 125, 83, 62, 50).

In all sessions, subjects participated in three practice rounds in which all subjects were assigned the same values and bid against two computerized bidders. The computerized bidders were programmed to submit predetermined bids to ensure that every subject to have the same learning experience. In the main experiment, a new set of random values were drawn in each auction. The change in the size of the budget after the first 10 rounds was announced at the end the 10th round.

In each auction, bidders were given 1 min (1.5 min for the first three auctions) to make decisions. Subjects were provided with starting capital balances of 500 experimental currency units (ECUs) with earnings added to this and losses subtracted from it. In the instructions, bidders were told that they could lose money, but no restrictions on bids were imposed to prevent this. In the case of bankruptcy, bidders would no longer be able to participate in the experiment and be asked to leave with the show-up fee. However, no bankruptcies occurred.

Following each auction, feedback was provided in the form of a table reporting values, bids, allocations, payments and earnings of all bidders. All values and earnings were denominated in ECUs. Final earnings were converted into dollars at the rate of 100ECUs=$1. In addition to this, subjects were paid a $4 show-up fee designed to give some money in case of bankruptcy. Earnings averaged $17.75 per subject in UPA and $17.36 in UPA-M, with sessions lasting on average 1.5 hours.

The experiment was run in the Ohio State University Experimental Economics Laboratory between Mar 2019 and Sep 2019. Subjects participating in the experiment were generally undergraduate students drawn from all disciplines and all were recruited through ORSEE (Greiner, 2004). Each subject participated in no more than one experimental session. The experiment was computerized, programmed using z-Tree (Fischbacher, 2007).

5. Experimental results

5.1 Efficiency and revenue

Table 1. Realized efficiency

	Session 1		Session 2		Session 3		All	
	UPA	UPA-M	UPA	UPA-M	UPA	UPA-M	UPA	UPA-M
w=100	76.7% (1.52%)	79.7% (1.51%)	74.8% (1.79%)	79.0% (1.63%)	74.1% (1.41%)	80.8% (1.31%)	75.2% (0.9%)	79.8% (0.9%)
w=250	82.9% (1.73%)	84.0% (1.61%)	80.3% (2.03%)	82.4% (1.69%)	78.9% (1.87%)	82.8% (1.82%)	80.8% (1.1%)	83.2% (1.0%)

* Parentheses are standard errors of the mean.

Table 1 shows realized average efficiencies of UPA and UPA-M Sessions. On average, UPA-M achieved higher efficiency than UPA under both w=100 and w=250. Under w=100, even the lowest efficiency in UPA-M (79.0%, Session2) is higher than the highest efficiency in UPA(76.7%, Session 1). As a result, Mann-Whitney test between UPA and UPA-M under w=100 turns out to be significant ($p<0.05$) using session averages as the unit of observation[7]. However, under w=250, the same test between UPA and UPA-M was not statistically significant ($p=0.14$). This is because not all sessions in UPA-M had higher efficiency than efficiencies in UPA; UPA-M session2 (82.4%) achieved lower efficiency than UPA session1 (82.9%). The different results between w=100 and w=250 implies that the relaxed budget rule in UPA-M more effectively improves the auction outcomes when the budget is tighter.

Table 2. Realized Revenue

	Session 1		Session 2		Session 3		All	
	UPA	UPA-M	UPA	UPA-M	UPA	UPA-M	UPA	UPA-M
w=100	20.6 (0.97)	30.8 (1.45)	21.4 (1.31)	23.3 (1.53)	21.6 (1.09)	28.3 (1.54)	21.2 (0.64)	27.6 (0.89)
w=250	45.0 (2.27)	43.7 (2.35)	42.7 (3.48)	37.9 (2.52)	40.6 (2.49)	47.8 (2.55)	42.8 (1.54)	43.2 (1.45)

* Parentheses are standard errors of the mean. The number are in ECUs

Table 2 shows realized average revenues between UPA and UPA-M Sessions. The revenue comparison between the two treatments are essentially the same with efficiency results. Under

[7] Regressions using all period outcomes with dummy variables for the two auction formats, with clustering at the session level yield essentially the same as the Mann Whitney test. (see online Appendix 4).

w=100, the lowest revenue in UPA-M (23.3, session 2) is higher than the highest revenue in UPA(21.6, session 3). As a result, Mann-Whitney test between UPA and UPA-M was significant ($p<0.05$) under w=100. However, the rank order of revenues under w=250 was mixed between UPA and UPA-M and the Mann-Whitney test was not significant($p=0.413$).

Table 3. frequencies of truthful bid.

		1^{st} share	2^{nd} share	3^{rd} share	4^{th} share	5^{th} share
w=100	UPA	19.5% (1.7%)	14.4% (1.5%)	9.6% (1.2%)	8.2% (1.2%)	6.8% (1.1%)
	UPA-M	24.6% (1.8%)	15.4% (1.5%)	7.7% (1.1%)	6.1% (1.0%)	4.9% (0.9%)
w=250	UPA	26.1% (1.8%)	25.3% (1.8%)	20.2% (1.7%)	12.3% (1.4%)	10.5% (1.3%)
	UPA-M	28.4% (1.9%)	25.8% (1.8%)	18.2% (1.6%)	12.5% (1.4%)	12.1% (1.4%)

* Parentheses are standard errors of the mean

Frequencies of truthful bid (= bidding one's true value) are closely related to the efficiency results. Under w=250, frequencies of truthful bid on each share were not much different between UPA and UPA-M sessions (Table 3 bottom two rows). This is in line with the insignificant efficiency difference between UPA and UPA-M sessions. Under w=100, frequencies of truthful bid on 2nd-5th shares between the two treatments are essentially the same (Table 3). However, bidders much more frequently submit their true values on the first share in UPA-M than UPA. This implies that the difference in efficiencies between the two treatments under w=100 results from the different bidding behavior on the first share.

Figure 1. frequencies of truthful bid on the first share by bidder's value

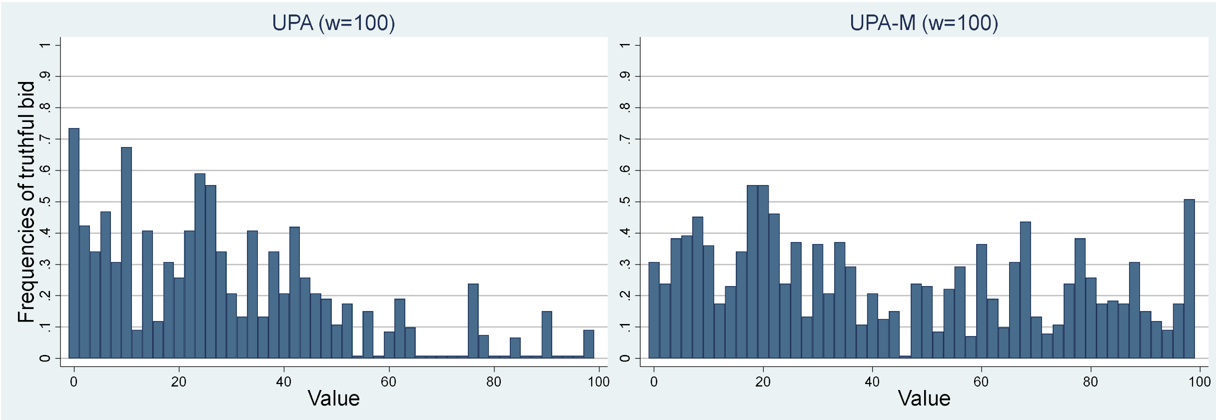
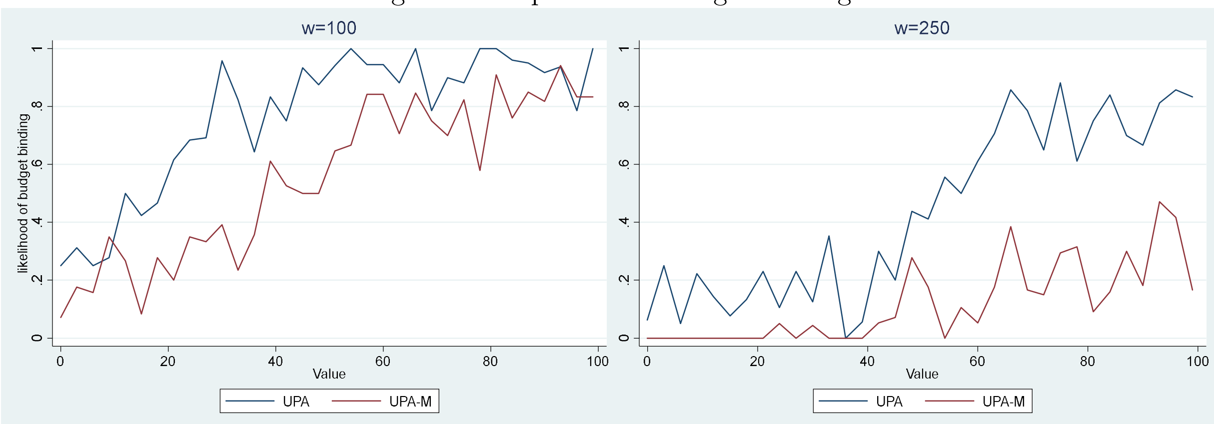


Figure 1 shows the frequencies of truthful bid on the first share between UPA/w=100 and UPA-M/w=100 across bidders' values. In UPA, bidders with high-values (v> 60) rarely submit their true values and this resulted in lower efficiency in UPA. [8] In UPA-M, in contrast, high-value bidders submit their true value about the same frequencies with the lower value bidders.

Figure 2. Frequencies of budget binding



[8] Further investigation in bidding behavior (chapter 5.2.1) shows that most high-value bidder submit bid lower than their values.

Figure 2 shows the frequencies of budget binding[9] in the two auction formats under both budgets. Under w=100, bidders with values above 40 mostly bound by the budget in UPA, and in UPA-M, bidders with values above 60 mostly bound by the budget. That UPA has a lower threshold of budget binding shows that the budget rule in UPA indeed more stringent than UPA-M, and this to some extent explains lower efficiency and revenue of UPA than UPA-M under w=100. In both auctions, under w=100, bidders are more often bound by the budget than corresponding w=250 sessions, and this resulted in poorer performance in w=100 sessions, in particular, the revenue. Under w=250, bidders with values below 50 seldom bound by the budget in both auctions. Bidders with values above 70 seems to be sometimes bound by the budget in both auctions. Although the frequencies of high-value bidders being bound by the budget are different in the two auctions, there is no noticeable difference in the threshold values where bidders begin to be constrained by the budget.

> *Result 1:* The realized outcomes were broadly consistent with the three hypotheses. First, bidders in UPA-M more frequently submit true value than UPA under w=100, in particular on the first share. Under w=250, there were no noticeable differences in the frequencies of truthful bidding between the two auctions. Second, bidders in UPA more frequently bound by the budget constraint than UPA-M. Under w=100, there were threshold values above which most bidders are constrained by the budget, and UPA had lower threshold than UPA-M. Under w=250, lower value bidders were rarely affected by the budget. Higher value bidders were constrained by the budget, but the threshold values where bidders begin to be constrained were not noticeably different between the two auctions. Third, as a result of above findings, the efficiency and revenue of UPA were lower than UPA-M under both w=100

[9] I counted the cases when bidders use more than 97% of the budget as budget binding. Specifically, In UPA, I counted when the sum of bids surmounts 97% of budget. In UPA-M, I counted when any bid surmounts 97% of the budget assigned for the bid.

and w=250. However, the differences were only significant under w=100. This implies that the lenient budget rule in UPA-M more effectively improves the auction outcomes when bidders facing tighter budget.

5.2 Bidding behavior

5.2.1. UPA sessions

Figure 3. Bid Plots in UPA Sessions

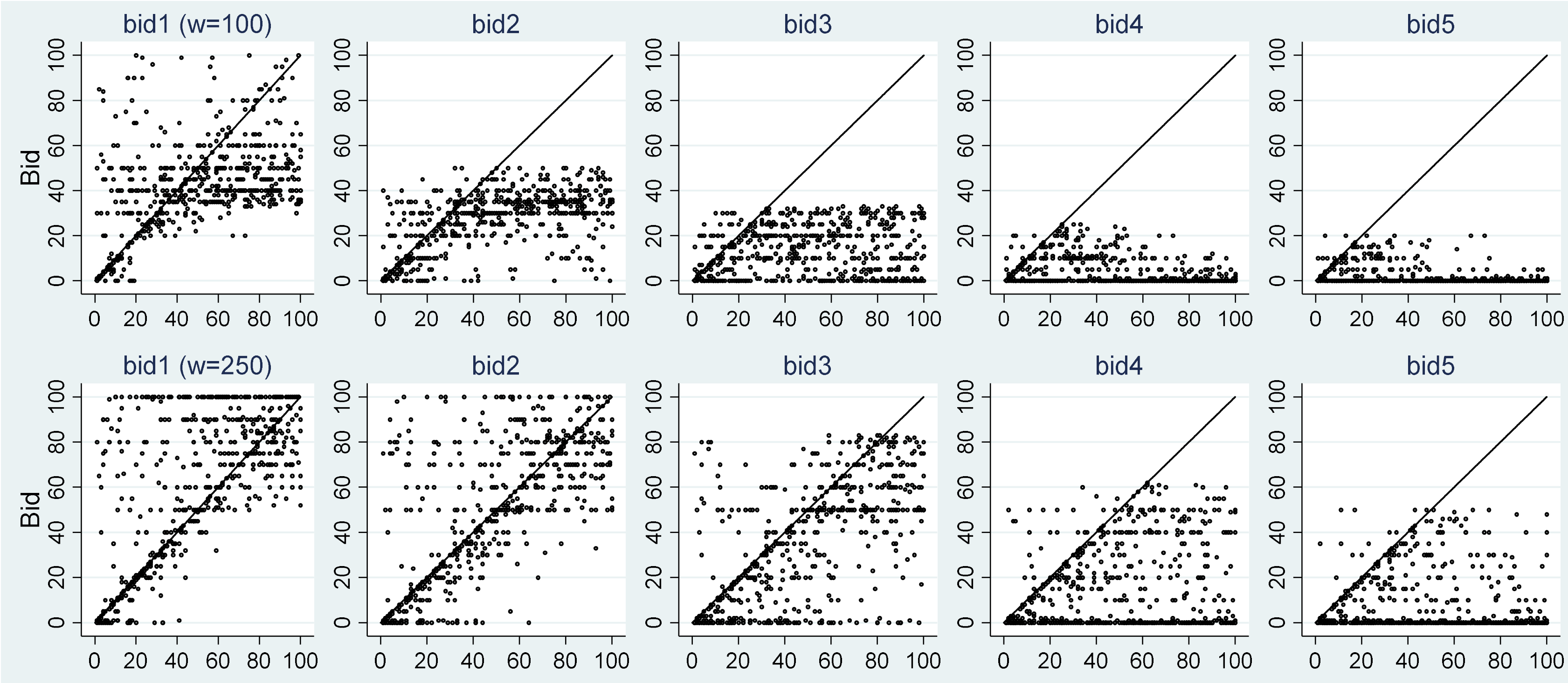


Figure 3 provide bid plots on the five shares in UPA. Upper panels are bids under w=100 and lower panels are bids under w=250. In UPA, submitting true value on the first share is *not* a dominant strategy, and the figure shows that a large number of bids indeed deviate from the true values under both w=100 and w=250 on the first share. However, the patterns of deviation are different between the two budgets. Under w=100, a dominant portion of deviation was mid to high-value bidders submitting bids lower than the true values (55.1% of bids were below value on the first share, table 5). An obvious reason for this bidding behavior is that high-value bidders saving budget on the first share and using the saved budget to bid on the second and later shares. Under w=250, a large number of bidders submit above their values (45.1% of bids were above value). Bidding above one's value is a dominated strategy but it is a common observation in multi-unit demand uniform price auction (Kagel and Levin, 2001).[10]

Theory predicts demand reduction for the second and later bids, and bidders indeed submit bid below values for second and later shares under both w=100 and w=250. However, it is important that not all bid below values are intended demand reduction in this experiment. Bidders are forced to reduce demand if they do not have enough budget to bid true value. For example, under w=100, a bidder with the true value 40 cannot submit 40 on the second share if she already submits 70 on the first bid; now the maximum she can bid on the second share is 30. After categorizing demand reduction (=underbid) into two cases, the budget exhausted case and the intended case, there are clear difference in the patterns of demand reduction between w=100 and w=250 treatments.

Table 5. Bidding in UPA sessions (w=100)

				Demand reduction			
w=100	True	Overbid	Under	Budget Exhausted	intended	Full demand reduction	Budget Exhausted
1st	19.5%	25.4%	55.1%	-	55.1%	1.2%	-

[10] Kagel and Levin (2001, 423-424p) reported a large number of bids above values in a multi-unit demand uniform price auction in spite of explicit instructions against bidding above value. In addition, Bidding above values is typical outcome in single-unit second price auction (Kagel and Levin, 1993; Kagel, Harstad and Levin, 1985)

2nd	14.4%	10.7%	74.9%	11.4%	63.5%	4.2%	0.9%[11]
3rd	9.6%	4.7%	85.6%	43.5%	42.1%	22.6%	12.8%
4th	8.2%	1.2%	90.5%	60.4%	30.2%	62.1%	44.9%
5th	6.8%	0.9%	92.3%	74.6%	17.7%	76.8%	60.0%

Table 6. Bidding in UPA sessions (w=250)

				Demand reduction			
w=250	TRUE	Overbid	Under	Budget Exhausted	intended	Full demand reduction	Budget Exhausted
1st	26.1%	45.1%	28.8%	-	28.8%	4.0%	-
2nd	25.3%	29.5%	45.3%	-	45.3%	7.2%	-
3rd	20.2%	13.7%	66.1%	7.54%	58.6%	15.3%	-[12]
4th	12.3%	4.2%	83.5%	25.61%	57.9%	40.7%	13.9%
5th	10.5%	1.9%	87.5%	40.70%	46.8%	59.1%	24.9%

Under w=100, demand reduction on the second share is mostly intended (63.5% / 74.9%, table 5): bidders submit bid below values even though they had enough budget to submit higher bid than what they submit. A reason of such intended underbid on the second shares would be saving budget for later shares as we can infer from underbidding on the first share. Another reason for underbidding is to set the market price low, which has been well documented in the uniform price auction literature. Higher frequencies of intended demand reduction on second share than the demand reduction on the first share reflects that both reasons are at work. For the third bid, bidders often constrained by the budget, and about half (43.5%/85.6%) of demand reduction is due to budget binding. For the fourth and fifth bids, budget binding becomes the dominant reason for demand reduction (60.4%/90.5%, 74.6%/92.3%). This shows strong effect of tight budget on the bidding behavior. Full demand reduction (= zero bids) was often observed for third-fifth shares (table5, column7). For third bids, about half of full demand reduction was due to budget exhausting (12.8%/22.6%). For fourth and fifth bids, a large number of bids (62.1%, 76.8%) are zero bids and they were mostly due to budget binding (44.9%/62.1%, 60.0%/76.8%). This too shows heavy effects of the tight budget on the bidding behavior.

[11] This only happens when a bidder submits 100 on the first share, which was extremely rare.
[12] Zero bid for third bid cannot be due to budget binding since bidders have 250.

Under w=250, demand reduction on the second share is all intended since bidders have enough budget to submit any bids for two. The frequencies of intended underbid on the second share in w=250 was much lower than the intended underbid in w=100 (45.3% vs 63.5%), and this reflects that bidders in w=250 have less incentive to submit lower bid to save budget for later shares. Indeed, bidders in w=250 more frequently submit true value or overbid on the second share than the bidders in w=100 (25.3%,29.5% vs 14.4%,10.7%). For the third bid, bidders are sometimes constrained by the budget but the frequency of budget binding is low (7.54%/66.1%). For the fourth and fifth bids a large number of bidders are constrained by the budget, but intended demand reduction is still the dominant reason for underbidding. Full demand reduction is often observed for third-fifth shares although the frequencies are uniformly lower than w=100 (column7, table 6). Many of such full demand reduction is due to budget binding, but this accounts for less than half of full demand reduction. This implies that bidder often fully reduced demand in order to set a low price in the auction as observed in previous uniform price auction literature (Kagel and Levin, 2001; List and Reiley, 2000).

In w=250, an interesting bidding behavior, which was absent under w=100, was observed: a strong overbidding behavior on the first share regardless of values (Figure 7 bottom left). A substantial number of bidders (17.2%) tender extreme overbids on the first share.[13] At the first glance, this seems to be irrational bidding behavior. However, this bidding behavior is not as risky as it seems. Since they submit only one (or at most two) such bids that have a small chance of crucially affecting the market price, it is unlikely that they wind up paying a higher price than their value: a low chance of getting negative payment sustains this type of overbidding.[14]

[13] I counted 98, 99, 100 as extreme bids.

[14] Among 17.2% who submitted extreme bids on the first share, 11.1% submitted such bids only on the first share and 6.1% submitted on the first and the second shares. The chance of losing money (negative profit) was 30.2% if submitting one extreme bid, and 31.4% if submitting two extreme bids.

Submitting very high price have been discussed in the literature of the uniform price auctions. Journals and academic research studies (Delaney and Sidel, 2004; Jagannathan et al., 2015) reported such incentive to bid high for a small number of units in initial public offerings that use uniform price auctions such as Goolge's IPO. [15] Levin (2005) suggested a theory that supports an equilibrium in which bidders submit very high price on earlier shares in a specific setting. [16] Kagel and Levin (2001) reported that bidders in uniform price auctions submit high bids for the first share in their experiment. The current experiment replicates and confirms extreme overbidding behavior in the uniform price auction.

Result 2. The bidding behavior in UPA between w=100 and w=250 were contrastingly different. For the first bid, in w=100, bidders mostly bid below values to save budget, while in w=250, bidders mostly overbid. The pattern of demand reduction was different as well. In w=100, bidders intended to reduce demand for earlier shares (1st-2nd) to save budget and are mostly forced to reduce demand for later shares (3rd-5th) due to the budget binding. However, in w=250, the frequencies of intended demand reduction for earlier shares were lower than w=100 since they had less incentive to save budget. For the later shares, on the other hand, bidders more frequently intended to reduce demand than w=100. Full demand reduction was often observed for third-fifth bids under both treatments, but the reasons behind were different. In w=100, most zero bids were due to bidders exhausting budget, while in w=250, bidders intended to fully reduce demand to set a low price. Lastly, in w=250, a large number of bidders submitted extremely high bid on the first share and sometimes on second share.

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[15] "… disruption can result when some bidders place noncompetitive (i.e. arbitrarily high-priced) bids. In a uniform price auction, such ''free riding'' places the bidder first in line for shares but may have little effect on the clearing price. However, each such bid reduces the pool of shares available to investors who actively participate in price discovery. …" (Jagannathan et al., 2015). "Because anyone bidding below the clearing price won't get any shares, there will be an incentive to bid high." (Delaney and Sidel, 2004, WSJ - Google IPO Aims to Change the Rules)

[16] Specifically, he suggested the equilibrium when the number of bidders and the items on offer are the same.

5.2.2 UPA-M sessions

Figure 4. Bid Plots in UPA-M Sessions

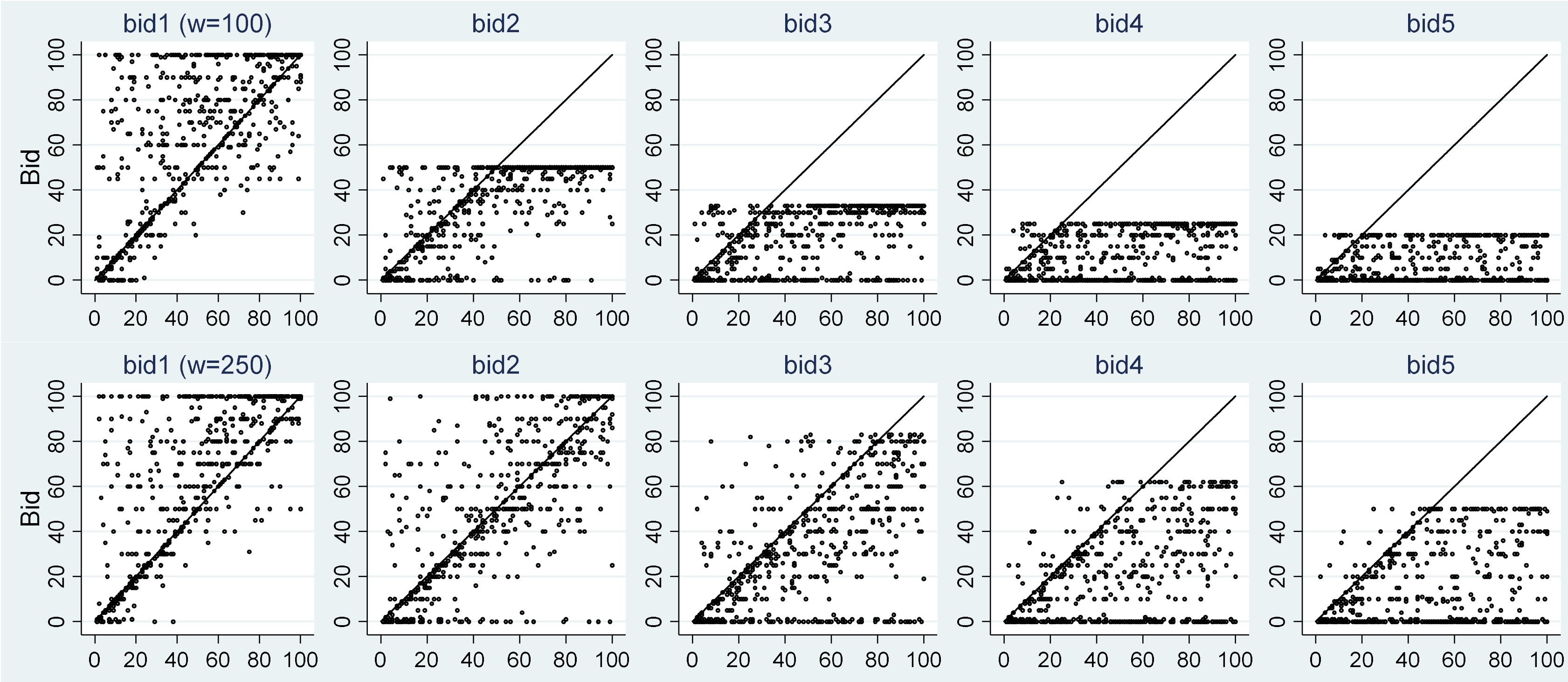

Figure 4 provides bid plots on the five shares in UPA-M. In UPA-M, theory predicts that bidders submit their true values on the first share. However, more than half of bidders submit above value on the first share under both w=100 and w=250 (54.0%, 56.1%, table 7,8, column 3). A large number of bidders submitted extreme overbid and this accounts for about half of the overbidding behavior (21.2%/54.0%, 25.6%/56.1%).[17] [18] About one-fifth (21.2%) bidders in UPA-M/w=100 submitting extreme overbids is in stark difference with UPA/w=100 where bidders rarely submitted such extreme overbid (1.2%). In addition, the frequencies of truthful bidding in UPA-M/w=100 is much higher than UPA/w=100 (24.6% vs 19.5%). These show that the relaxed budget rule in UPA-M removed the incentive to bid lower on the first share to save budget. The frequencies of bidding one's true value on the first share are essentially the same between w=100 and w=250 (24.6% vs 28.4%). And those frequencies are also similar to UPA/w=250 (26.1%)

Theory predicts demand reduction for the second and later bids, and bidders indeed underbid on second and later shares under both w=100 and w=250. However, not all underbidding is intended demand reduction. For example, under w=100, a bidder with the true value 60 cannot submit 60 on the second share since the second bid is capped by 50 due to the budget rule. Table 7 and 8 distinguish those capped bids[19] and intended demand reduction. After categorizing demand reduction into the two (capped and intended) categorizes, the frequencies of intended demand reduction are essentially the same between w=100 and w=250 (column 6 of table 7 and 8). This is in clear contrast with UPA sessions in which intended demand reduction patterns are different between w=100 and w=250. In UPA/w=100, bidders have strong incentive to save budget for later shares and this incentive make the demand reduction pattern different from UPA/w=250. Absent of such incentive, bidders in UPA-M have only a common incentive to reduce bid (to set a low price) in both w=100 and 250, and the same pattern of demand reduction reflects this.

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[17] I counted 98, 99, 100 as extreme bids.

[18] As in UPA/w=250, the chance of losing money was low despite such extreme bids. 9.0% of bidders ended up losing money under w=100 and 21.9% under w=250 by submitting extreme bids.

[19] I counted bids as capped bids (=unintended demand reduction) if a bid is below value but not below 97% of bid cap assigned for the bid. In figure 4, these capped bids are pooled at the top right side for second-fifth bids.

Table 7. Bidding in UPA-M sessions (w=100)

				Demand reduction		
w=100	True	Overbid	Under	Capped	Intended	Full demand reduction
1st	24.6%	54.0%	21.4%	-	21.4%	3.7%
2nd	15.4%	13.2%	71.4%	32.1%	39.3%	11.2%
3rd	7.7%	5.3%	87.0%	22.3%	64.7%	22.8%
4th	6.1%	3.2%	90.7%	18.2%	72.5%	39.1%
5th	4.9%	2.3%	92.8%	19.3%	73.5%	50.0%

Table 8. Bidding in UPA-M sessions (w=250)

				Demand reduction		
w=250	True	Overbid	Under	Capped	Intended	Full demand reduction
1st	28.4%	56.1%	15.4%	-	15.4%	2.6%
2nd	25.8%	30.0%	44.2%	-	44.2%	11.1%
3rd	18.2%	11.1%	70.7%	1.6%	69.1%	20.4%
4th	12.5%	5.3%	82.3%	3.2%	79.1%	34.6%
5th	12.1%	2.8%	85.1%	7.2%	77.9%	44.7%

Full demand reduction is often observed on third-fifth shares. Since bidders in UPA-M have non-zero bid caps for each bid they submit, all zero-bids are intended demand reduction to set the price low. The patterns of full demand reduction between w=100 and w=250 are essentially same. Compared to UPA sessions, the frequencies full demand reduction in UPA-M are noticeably lower for later shares. In UPA/w=100 sessions, 62.1% and 76.8% are zero-bids on fourth and fifth shares respectively, while in UPA-M/w=100 sessions, the corresponding frequencies are 39.1% and 50%. In UPA/w=250 sessions, 59.1% were zero-bids on the fifth share while the number is 44.7% in UPA-M/w=250 sessions. This shows that the relaxed budget rule in UPA-M helped bidders to submit meaningful non-zero bids for later shares.

Results 3: Bidding behavior in UPA-M between w=100 and w=250 were essentially identical except bidders in w=100 more often capped by the budget rule. For the first bid, under both budgets, more than half of bidders submit bids above values and about half of such bids were extreme overbid. This is in contrast to UPA/w=100 session in which bidders mostly underbid and rarely submit extreme bids, and shows that the relaxed budget rule removed bidders' incentive to bid below values to save budget. The pattern of intended demand reduction was essentially identical between w=100 and w=250. So are full demand reduction patterns between the two budgets. These implies that the motivation behind demand reduction between the two budgets is a common one, which is to set the market price low.

6. Discussion

This paper experimentally studies the effects of two budget rules in the uniform price auction, one conventionally adopted in practice and another suggested in the current paper. The conventional budget rule employed in practice such as in Google's IPO and Korea's emission trading auction restricts bidders from submitting bids that adds up to more than the budgets of bidders. Although this rule is straightforward and easy to implement, the rule is unnecessarily restrictive. The alternative rule suggested in the current paper restricts bidders kth bid from exceeding the budget divided by k. This rule seems to be complicated but is still implementable[20] and much lenient than the conventional rule.

The experimental results show that the alternative rule uniformly improve efficiency and revenue of the uniform price auction under both weak and strong budget constraints. The improvement was much pronounced in the strong budget as the conventional rule heavily distorts high-value bidders' bidding behavior; they strongly underbid even on the first share in order to save budget and bid on later shares. In contrast, the improvement was only marginal and not statistically significant under the weaker budget; the efficiency and revenue losses of the conventional rule compared to the alternative rule are less than 3%. This together with the simplicity of the conventional rule explains the wide use of such rule in practice.

[20] The auctioneer can ask security deposit that equals to $\max_k (b_i^k \times k)$

The current study can be extended in several ways. The most obvious way would be to explore more flexible and practical environments such as common-value or affiliated value setting, increased number of bidders and shares, heterogenous budget constraints, decreasing marginal utility or complementarity. However, a more interesting further study would be investigating external validity of the results of this study. To this end, further research may investigate how often bidders are budget constrained in uniform price auctions in practice and how large a typical budget size of a bidder compared the total value of items on sale. A large-scale field experiment using the alternative rule would be also interesting future study.

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Appendix

A.1 Characterization of an equilibrium in UPA

Let $\boldsymbol{\beta}^{UPA}(v,w)\big(\beta^1(v,w),\beta^2(v,w),\dots,\beta^m(v,w)\big)$ denote a symmetric equilibrium of UPA. Let $\boldsymbol{c}_{-\boldsymbol{i}} = (\beta_1^1,\dots,\beta_1^m,\dots,\beta_{i-1}^1,\dots,\beta_{i-1}^m,\beta_{i+1}^1,\dots,\beta_{i+1}^m,\dots,\beta_n^1,\dots,\beta_n^m)$ be the competing bids facing bidder *i* and the distribution of the random variable $\boldsymbol{C}_{-\mathbf{i}}$ has a density given by $h(\boldsymbol{c})$. Then, let c_{-i}^k be *k*th highest element among $\boldsymbol{c}_{-i}$. An equilibrium in UPA is acquired by solving the following equation.

$$\boldsymbol{\beta}^{UPA}(v_i,w) = \underset{(b^1,\dots,b^m)\in\{(b^1,\dots,b^m)|\sum_{k=1}^m b_i^k\le w\}}{\text{argmax}} \int_{\{\boldsymbol{c}_{-i}\,:\,c_{-i}^1<b_i^m\}} m(v_i - c_i^1)h(\boldsymbol{c})\boldsymbol{dc}$$
$$+ \int_{\{\boldsymbol{c}_{-i}\,:\,c_{-i}^2<b_i^{m-1}\ and\ c_{-i}^1> b_i^m\}} (m-1)(v_i - max\{\ c_{-i}^2,\ b_i^m\})h(\boldsymbol{c})\boldsymbol{dc}$$
$$+ \quad \dots\dots$$
$$+ \int_{\{\boldsymbol{c}_{-i}\,:\,c_{-i}^m<b_i^1\ and\ c_{-i}^{m-1}> b_i^2\}} (v_i - max\{\ c_{-i}^m,\ b_i^2\})h(\boldsymbol{c})\boldsymbol{dc}$$

Similarly, an equilibrium of UPA-M is characterized by substituting $(b^1,\dots,b^m)\in\{(b^1,\dots,b^m)|\forall k\le M, b^k\le w/k\}$ to the budget constraint restriction.

A.2 An example of UPA equilibrium with a budget constraint

A seller sells two equal shares and there are two bidders (1,2). Each bidder has a private value and the value is the same for the two shares. Bidder *i*'s private value v_i is independently drawn from a uniform distribution [0,1]. Each bidder can submit up to two bids and the third highest bid sets the market price. The two bidders have a common budget constraint w.

The following strategy constitutes an equilibrium for any w: bidder *i* submits $b_i^1 = \min(v_i, w)$ for the first share and submit $b_i^2 = 0$ for the second share. Both bidders each win one share and pay 0. Both bidders do not have an incentive to deviate from this strategy assuming the other bidder sticks to this strategy. For the first bid, it is dominated to bid more than v_i and it is not possible to bid more than w if constrained. There is also no incentive to reduce the first bid since reducing it only decreases the chance of getting the first share and the saved budget by reducing the first bid will never be used for the second bid as the second bid is 0. For the second bid, increasing the second bid is not profitable. If bidder 1 increases his second bid, this surely increases the market price he pays, which directly reduce his payoff. But bidder 1 wins the second share only if he increases his bid second bid enough to beat b_2^1 (the first bid of the second bidder). It can be easily shown that the first effect is greater than the second effect (as in Ausubel et al., 2014). Thus, the strategy profile is an equilibrium.

A3. Theoretical predictions in UPA, UPA-M.

Proposition 1. In UPA and UPA-M, bidding higher than v_i for a share is dominated by bidding v_i. In addition, In UPA-M it is a dominant strategy to bid v_i for the first share, while it is not a dominant strategy in UPA.

Proof. Bidding higher than v_i gives more winning shares than bidding v_i only when the market price is above v_i, in which case bidder i does not want to win more shares. Thus, bidding above one's value is dominated by bidding v_i

In UPA-M, bidding less than v_i on the first share only reduces the chance of winning the share but never lower the market price that bidder i has to pay (since his first bid will never be the market price if he wins the first share). Thus, bidding the true value on the first share is a dominant strategy. Note that in UPA-M, bidding on the first share does not have any effects on bids for the later shares since the budget rule is applied to each bid. In UPA, however, the budget rule applies to the sum of all bids, so it could be profitable to bid less than value on the first share to save budget and use the saved budget to submit meaningful bids for later shares.

Regression analysis for efficiency and revenue

Efficiency and revenue with auction format dummy variables

VARIBLES	Efficiency (w=100)	Efficiency (w=250)	Revenue (w=100)	Revenue (w=250)
Period	0.002	0.006**	0.040	-0.095
	(0.001)	(0.002)	(0.163)	(0.342)
UPA-M	0.046***	0.024	6.479**	0.379
	(0.009)	(0.012)	(2.022)	(2.741)
Constant	0.739***	0.772***	20.941***	43.311***
	(0.014)	(0.017)	(0.807)	(2.703)
Observations	380	380	380	380
R-squared	0.038	0.024	0.085	0.000

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

This table shows regression results of efficiency and revenue with dummy variables for auction formats, clustering errors at session level. The base treatment is UPA. While testing differences between dummy variables I used F-test.

Hypothesis 3 predicts UPA-M achieves higher efficiency and revenue than UPA. Under w=100, UPA-M achieved 4.7% higher efficiency and 6.479 higher revenue. Both differences were statistically significant ($p<0.05$). Under w=250, UPA-M achieved 2.4% higher efficiency and 0.379 higher revenue. However, both differences are slight and not statistically significant. This implies that the improvement of UPA-M due to its relaxed rule is only effective when the budget constraint is tight.